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HEAT TRANSFER OF A CYLINDRICAL FILM-TYPE TRANSDUCER  
IN A HOT-WIRE ANEMOMETER WITH INTERNAL HEATING

I. L. Povkh, F. V. Nedopekin,  
and A. M. Novikov

UDC 533.6.08:536.5

The article examines the heat transfer of a cylindrical film-type transducer in a hot-wire anemometer with an internal heat source operating in a moving medium.

One of the more promising directions being taken in the development of thermoanemometric measurement technology, employed in experimental aerohydrodynamics, is the use of transducers in the form of a dielectric base with a superimposed metallic film [1, 2]. Theoretical and experimental studies show that the process of heat exchange between the film and the dielectric base has a significant effect on the metrological characteristics of the transducer [3]. Heat exchange between the film and base can be reduced by using an internal heat source to create a temperature gradient between them. To analyze the operation of such a transducer, we will examine a model in the form of a finite cylinder of length  $2l$  with a surface heat source of length  $2n$ . An internal heat source  $\omega$  is located in the axis of the cylinder (Fig. 1). Since the local heat-transfer coefficient of the heated cylinder, immersed in the flow of the medium, depends on the angle  $\varphi$ , then its heat-conduction equation has the form [4]

$$a \left[ \frac{\partial^2}{\partial r^2} T(r, z, \varphi) + \frac{1}{r} \frac{\partial}{\partial r} T(r, z, \varphi) + \frac{\partial^2}{\partial z^2} T(r, z, \varphi) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} T(r, z, \varphi) \right] + \frac{\omega}{c\gamma} = 0. \quad (1)$$

However, the temperature field of the substrate under the surface heat source is nearly axis-symmetrical, and Eq. (1) takes the form

$$a \left[ \frac{\partial^2}{\partial r^2} T(r, z) + \frac{1}{r} \frac{\partial}{\partial r} T(r, z) + \frac{\partial^2}{\partial z^2} T(r, z) \right] + \frac{\omega}{c\gamma} = 0. \quad (2)$$

The boundary conditions

$$-\lambda \frac{\partial}{\partial r} T(R, z) + g - \alpha [T(R, z) - T_{\text{md}}] = 0, \quad (3)$$

$$T(0, z) \neq \infty, \quad (4)$$

$$\frac{\partial}{\partial z} T(r, 0) = 0, \quad (5)$$

$$\frac{\partial}{\partial z} T(r, n) = 0. \quad (6)$$

Using (5) and (6), we execute a finite Fourier cosine transform with respect to the coordinate  $z$  [5]:

$$r \frac{d^2}{dr^2} T(r, p) + \frac{d}{dr} T(r, p) - rp^2 T(r, p) + r \frac{\omega l \sin \frac{p\pi c}{l}}{c\gamma a p \pi} = 0 \quad (7)$$

and set the boundary conditions:

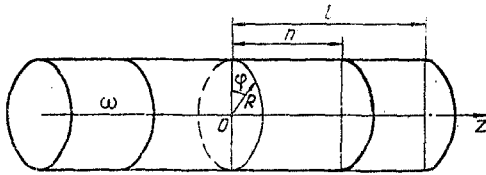


Fig. 1. Model of cylindrical film-type transducer of a hot-wire anemometer with internal heating.

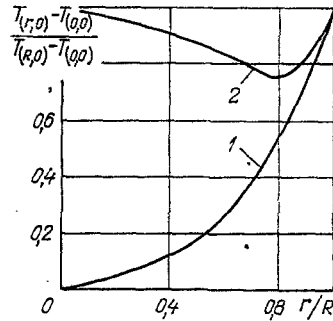


Fig. 2

Fig. 2. Temperature distribution over the radius of the cylinder: 1, 2) with and without internal heat sources, respectively.

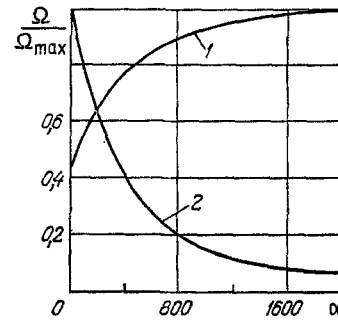


Fig. 3

Fig. 3. Dependence of optimum power of internal heat source on coefficient of convective heat transfer for constant-temperature (a) and constant-current (2) hot-wire-anemometer transducers.  $\alpha$ ,  $W/m^2 \cdot K$

$$\alpha \left[ T(R, p) - T_{md} \frac{l \sin \frac{p\pi n}{l}}{p\pi} \right] + \lambda \frac{dT(R, p)}{dr} = \frac{gl \sin \frac{p\pi n}{l}}{p\pi}, \quad (8)$$

$$T(0, p) \neq \infty. \quad (9)$$

With allowance for (8) and (9), the solution of Eq. (7) has the form

$$T(r, p) = \frac{l \sin \frac{p\pi n}{l}}{p\pi} \left\{ \frac{g + \alpha T_{md}}{\alpha \left[ \frac{I_0(Rp)}{I_0(rp)} + \lambda p \frac{I_1(Rp)}{I_0(rp)} \right]} + \frac{\omega}{c\gamma a p^2} \left[ 1 - \frac{1}{\frac{I_0(Rp)}{I_0(rp)} + \lambda p \frac{I_1(Rp)}{I_0(rp)}} \right] \right\}. \quad (10)$$

Performing an inverse Fourier transform and introducing  $Bi = \alpha R/\lambda$ , we obtain

$$T(r, z) = \frac{n}{l} \left[ \frac{g + \alpha T_{md}}{\alpha} - \frac{\omega}{2c\gamma a} \right] + 2 \sum_{p=1}^{\infty} \left\{ \frac{g + \alpha T_{md}}{\alpha \left[ \frac{I_0(Rp)}{I_0(rp)} + \frac{pR}{Bi} \frac{I_1(Rp)}{I_0(rp)} \right]} + \frac{\omega}{c\gamma a p^2} \left[ 1 - \frac{1}{\frac{I_0(Rp)}{I_0(rp)} + \frac{pR}{Bi} \frac{I_1(Rp)}{I_0(rp)}} \right] \right\} \frac{\sin \frac{p\pi n}{l} \cos \frac{p\pi z}{l}}{p\pi}. \quad (11)$$

It is apparent from the curve (Fig. 2), constructed from Eq. (11), that the internal heat source reduces the temperature gradient between the surface heat source and the cylinder. If the temperature at the periphery of the cylindrical substrate is equal to the temperature of the film element, heat flow from the latter into the substrate is cut off and thus no longer has an effect on the metrological characteristics of the transducer. This is the case when the power of the internal heat source is optimum

$$\Omega = \left[ T(R, z)_{md} c\gamma a - \frac{g c\gamma a}{\alpha} \left\{ \frac{c}{l} + \frac{2l}{c} \sum_{p=1}^{\infty} \frac{\sin^2 \frac{p\pi c}{l}}{\pi^2 p^2 \left[ 1 + \frac{p}{Bi} \frac{I_1(Rp)}{I_0(Rp)} \right]} \right\} \right] \times$$

$$\times \left[ \frac{2l}{c} \sum_{p=1}^{\infty} \frac{I_1(Rp) \sin^2 \frac{p\pi c}{l}}{\pi^2 p^3 \text{Bi} I_0(Rp) \left[ 1 + \frac{p}{\text{Bi}} \frac{I_1(Rp)}{I_0(Rp)} \right]} - \frac{c}{2l} \right]^{-1}. \quad (12)$$

It depends on the coefficient of convective heat transfer of the transducer (Fig. 3), which may vary broadly during the measurements.

Thus, internal heating of a cylindrical film-type transducer in a hot-wire anemometer is one of the most effective means of improving its metrological characteristics.

#### NOTATION

$T(r, z, \varphi)$ , temperature of transducer substrate, °K;  $r, z, \varphi$ , coordinates of a point in the cylindrical system;  $a$ , diffusivity,  $\text{m}^2/\text{sec}$ ;  $c$ , specific heat,  $\text{J}/\text{kg}\cdot\text{deg}$ ;  $\gamma$ , density of the body,  $\text{kg}/\text{m}^3$ ;  $\omega$ , specific power of the internal heat source,  $\text{W}/\text{m}^3$ ;  $\alpha$ , coefficient of convective heat transfer,  $\text{W}/\text{m}^2\cdot\text{K}$ ;  $\lambda$ , thermal conductivity of substrate,  $\text{W}/\text{m}\cdot\text{K}$ ;  $g$ , quantity of heat released per unit of time per unit of surface of the film by the current passing through it,  $\text{J}/\text{sec}\cdot\text{m}^2$ ;  $T_{\text{md}}$ , temperature of the medium flowing around the cylinder, °K;  $p$ , Fourier transform parameter;  $I_0(Rp)$ ,  $I_0(rp)$ ,  $I_1(Rp)$ ,  $I_1(rp)$ , modified Bessel functions.

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#### COMPARISON OF MAXIMUM COEFFICIENTS OF HEAT TRANSFER TO A SURFACE SUBMERGED IN A FLUIDIZED BED WITH AN ESTIMATE OBTAINED FROM AN EMPIRICAL FORMULA

A. P. Baskakov and O. M. Panov

UDC 66.096.5

Empirical coefficients of heat transfer from a fluidized bed to bodies of various shapes are compared with coefficients calculated from a formula which considers heat transfer by particle and gas convection and by radiation.

As is known, heat transfer occurs between a fluidized bed and a body immersed in it by three different mechanisms: particle convection (this mechanism is sometimes referred to as conduction), gas convection, and radiation. These mechanisms are interrelated, but they are most often analyzed individually, permitting additivity, in order to simplify analysis and compare theoretical formulas. Given the current level of our knowledge of the complex process of heat transfer in fluidized systems, this approach is obviously the only proper approach.

The maximum heat-transfer coefficient  $\alpha_{\text{max}}$  is of the greatest practical interest. This value is reached at the optimum fluidization velocity  $w_{\text{opt}}$ . The dependence of  $\alpha$  on  $w$  has a maximum with a fairly gentle slope, so the heat-transfer coefficient changes little at velocities above the optimum value. It was noted in [1] that the difference  $w_{\text{opt}} - w_c \approx 0.35$  m/sec for most particles. In a bed of coarse particles, this difference is not large compared to the critical velocity  $w_c$ , and the optimum velocity turns out to be close to the free-falling velocity only in a bed of particles finer than 0.1 mm. However, in this case the dependence of the heat-transfer coefficient on the fluidization velocity (before it reaches

S. M. Kirov Ural Polytechnic Institute. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 45, No. 6, pp. 896-901, December, 1983. Original article submitted June 9, 1982.